## **AoPS Academy Magazine - Online Solutions**

## **Broomstick Challenge**

1. Take a long uniform object, like a meter stick or a cardboard cylinder from a wrapping paper roll. Hold it on two fingers, one on each edge of the object. Start moving your fingers towards each other. Where do your fingers meet? Try moving only your left finger and keep the right one stationary. What happens?

Your fingers will always meet in the center! If you try to only move your left finger, for example, you'll wind up pushing the cylinder and making it slide over your right finger until your fingers meet in the center.

2. Try putting your left finger near the center of the object and your right finger all the way out on the edge. Which finger has to support more weight? Using this observation, can you explain the result you found in part (1)?

The finger near the center will support a lot more weight. You can feel it pushes upward with a lot more force. That also means there's a lot more friction on that finger. So the other finger, which is far from the center, has only a little friction and can slip easily along the object, while the finger closer to the center is held in place. That's why when you start with your hands at opposite ends and start bringing them toward the center, whenever one gets more than an inch or two closer to the center than the other, it starts to feel more friction and can't slip until the other finger also gets closer to the center.

3. Try this again with a broomstick, or add some extra weight to one side of your meter stick. Where do your fingers meet now?

Your fingers still always meet at a particular point on the broom, but it's closer to the side of the broom with all the bristles. That side is heavier, and a finger down there needs to support more weight. So a finger on the side with the bristles is less likely to slip and doesn't move as far before your fingers meet. The place where your fingers meet is called the center of mass of the broomstick.

4. Suppose you wanted to hang your broomstick from a rope and have it suspended horizontally. Where should you attach the rope to the broomstick?

Attach the rope at the same place your fingers meet! Once your fingers come together, they're acting like a single point of support, just like a rope does. So a rope tied to the center of mass of the broomstick will let it hang horizontally.

5. If you were to cut the broomstick in half at the point where your fingers meet, which side would be heavier — the side with the broom, or the side that's all handle?

They're not the same! The side with the broomstick will be heavier. What makes something tip over when it's supported from a single point? It's a combination of the weight and the distance from the point of support. The two sides of the broomstick balance, but the side with the handle is further, on average, from the center of mass, so it must weigh less to compensate.

